Review of the convergence of some Krylov subspaces methods for solving linear systems of equations with one or several right hand sides

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Abstract

Krylov subspace methods are widely used for the iterative solution of a large variety of linear systems of equations with one or several right hand sides.

In this talk, we will derive new bounds for the GMRES method of Saad and Schultz, for solving linear system. We will give a sample formula for the norm of the residual of GMRES based on the eigenvalue decomposition of the matrix and the right hand side. This formula allows us to generalize the well known result on the convergence behavior of GMRES when the matrix has a full set of eigenvectors. The explicit formula of the residual norm of the GMRES when applied to normal matrix, which is a rational function, is given in terms of eigenvalues and of the components of the eigenvector decomposition of the initial residual. By minimizing this rational function over a convex subset, we obtain the sharp bound of the residual norm of the GMRES method applied to normal matrix, even if the spectrum contains complex eigenvalues. Hence we completely characterize the worst case GMRES for normal matrices. We use techniques from constrained optimization rather than solving the classical min-max problem (problem in polynomial approximation theory)

Known as one of the best iterative methods for solving symmetric positive definite linear systems, CG generates as FOM an Hessenberg matrix which is symmetric then triangular. This specific structure may be really helpful to understand how does behave the convergence of the conjugate gradient method and its study gives an interesting alternative to Chebyshev polynomials. The talk will deals also about some new bounds on residual norms and error A-norms using essentially the condition number. We will show how to derive a bound of the A- norm of the error by solving a constrained optimization problem using Lagrange multipliers.

This is a joint work with Mohammed Bellalij from university of Valenciennes, France