Computational methods for model reduction in large-scale nonlinear dynamical systems

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This project will be devoted to the development of new numerical methods for solving large scale linear and nonlinear systems of differential equations by using Krylov subspace techniques. These problems are related to dynamical systems which are important from theoretical point of view, but also for the applications itself such as the feedback stabilization and eigenvalue assignment problem for the large-scale size dynamical systems. The problem of the optimal control of large-scale differential systems is a widely investigated problem. In the context of dynamical systems, the behavioural equations are often differential linear and nonlinear equations. In the linear time-invariant case, many different techniques have been proposed these last years, among them moment matching methods or balanced trun- cation. They are based on the minimisation of the error tranfer function using two well norms: H_∞ and H_2. A theory which will fully describe an optimal control of models which are widely used to simulate, and/or optimize industrial, economical, and biological processes. For that purpose it is of our interest to describe some of the important system properties such as optimality of the solutions of the obtained linear and nonlinear matrix equations, including controllability, observability, feedback equivalence, and minimality; stability via Lyapunov, as well as input/output behaviour. Using projection methods such as Krylov subspaces, one obtain linear or non- linear matrix equations such as Lyapunov, Sylvester or Riccati equations. We plan to develop new algorithms based on the block versions of the well known Arnoldi algorithm. We will also derive a theory such as upper bounds for the norms of the errors. This project will mainly be devoted to the different norm optimization as well as the feedback stabilization for the large-scale dynamical systems. The main model which will be considered within this project is the time invariant nonlinear dynamic system. These dynamical systems come generally from the discretisation (in the space) of a time-varying (linear or nonlinear) Partial Differential Equations (PDEs). They are very important and play a fundamental role in model order reduction techniques. This problem is also related to differential algebraic Riccati equations used to minimize some energy or for stabilizing the dynamical system. Some methods have been already proposed and studied by K. Jbilou in many published papers in international scientific journals such as Linear Algebra and its Applications, Applied Numerical Mathematics and others.

The related Differential Matrix Riccati Equations (DMREs) play an important role in control theory, for example, in optimal control, filtering and esti- mation, decoupling and order reduction, etc.

Large-scale DMREs result from semi-discretized optimal control problems for instationary partial differential equations (PDEs) of parabolic or hyperbolic type, such as the heat equation, convectiondiffusion equations, or Burgers equations etc. Imposing a quadratic cost functional, the solution of the optimal control is often given via feedback control where the feedback operator is given in terms of an operator-valued differential matrix Riccati equation. Among the large variety of linearquadratic problems that can be solved if an efficient DMRE solver is available, the task of solving large-scale DMREs becomes an important issue in nonlinear optimal control or stabilization problems for a class of nonlinear PDEs.

The subject is to study and propose new techniques for nonlinear model reduction dynamical systems coming from some PDE's with a control. The two aspects will be important here : the theoretical part such as upper bound for the norms of the errors and for the practical part, the new Phd student should provide a useful package containing all the developed methods.

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