GRPIA: a new algorithm for computing interpolation polynomials

A. Messaoudi*

En collaboration avec M. Errachid, K. Jbilou et H. Sadok,
Calais, 15 Juin 2018

Abstract

Let $x_0, x_1, \cdots, x_n$, be a set of n+1 distinct real numbers (i.e., $x_m \neq x_j$, for $m \neq j$) and let $y_{m,k}$, for $m = 0, 1, \cdots, n$, and $k = 0, 1, \cdots, r_m$, with $r_m \in I\!N$, be given real numbers. It is known that there exists a unique polynomial p_{N-1} of degree $N-1$ with $N = \sum_{m=0}^{n}(r_m + 1)$, such that $p_{N-1}^{(k)}(x_m) = y_{m,k}$, for $m = 0, 1, \cdots, n$ and $k = 0, \cdots, r_m$. p_{N-1} is the Hermite interpolation polynomial for the set $\{(x_m, y_{m,k}),\ m = 0, 1, \cdots, n,\ k = 0, 1, \cdots, r_m\}$. The polynomial p_{N-1} can be computed by using the Lagrange generalized polynomials. Recently Messaoudi et al. in [**?**] presented a new algorithm for computing the Hermite interpolation polynomial called the Matrix Recursive Polynomial Interpolation Algorithm (MRPIA), for a particular case where $r_m = \mu = 1$, for $m = 0, 1, \cdots, n$. In this paper we will give a new formulation of the Hermite polynomial interpolation problem and derive a new algorithm, called the Generalized Recursive Polynomial Interpolation Algorithm (GRPIA), for computing the Hermite polynomial interpolation in the general case. A new result of the existence of the polynomial p_{N-1} will also be established, cost and storage of this algorithm will also be studied, and some examples will be given.

*Ecole Normale Supérieure, Mohammed V University in Rabat, Maroc. E-mail: abderrahim.messaoudi@gmail.com