Journées "Analyse Appliquée en Hauts-de-France"
1er et 2 Juin 2023
Amphi C001, Centre Universitaire de la Mi-Voix, Calais.

Program: June 1st

- 10h00-11h00: Min Chen (Purdue University)

  Title: Numerical investigation of water wave equations.

  Abstract. Studying water wave equations is a fascinating subject. In this talk, I'll look at the problems with numerical simulation and study the Boussinesq systems. Problems such as the global existence of solutions, the oblique interactions of line solitary waves and stability of various waves will be investigated.

- 11h15-12h15: Olivier Goubet (Laboratoire Paul Painlevé, Université de Lille)

  Title: Analyticity of the Global Attractor for Damped Forced Periodic Korteweg-de Vries Equation.

  Abstract. Damped forced KdV equations posses a global attractor. It is known for years that if the forcing term is smooth (say in any Sobolev space H^k) then solutions in the global attractor are smooth. What about the analyticity of the solutions if the forcing term is analytic ? A positive answer has been conjectured by Oliver and Titi two decades ago. We prove that the answer is positive and give some applications of this result.

12h30-14h00: Lunch

- 14h30-15h30: Bing-Yu Zhang (University of Cincinnati)

  Title: Well-posedness and Critical Index Set of the Cauchy Problem for the Coupled KdV-KdV Systems.

  Abstract. In this talk we will discuss the well-posedness of the Cauchy problem for the coupled KdV-KdV systems

$$\left\{\begin{aligned} u_t + a_1 u_{xxx} &= c_{11}uu_x + c_{12}vv_x + d_{11}u_xv + d_{12}uv_x, \\ v_t + a_2 v_{xxx} &= c_{21}uu_x + c_{22}vv_x + d_{21}u_xv + d_{22}uv_x, \\ (u,v)_{|t=0} &= (u_0, v_0) \end{aligned}\right. \tag{1}$$

  We consider this system posed on the real line $\mathbb{R}$ in the space $\mathcal{H}_0 := H^s(\mathbb{R}) \times H^s(\mathbb{R})$ and posed on the periodic domain in the following four spaces

$$\mathcal{H}_1^s := H_0^s() \times H_0^s(), \qquad \mathcal{H}_2^s := H_0^s() \times H^s(),$$
$$\mathcal{H}_3^s := H^s() \times H_0^s(), \qquad \mathcal{H}_4^s := H^s() \times H^s().$$

  It is assumed that $a_1a_2 \neq 0$ and at least one of the coefficients for the quadratic terms is not 0.

  For each $k \in \{0,1,2,3,4\}$, we show that for any given a_1, a_2, (c_{ij}) and (d_{ij}), there exists a unique critical index $s_k^* \in (-\infty, +\infty]$ such that the system (1) is locally analytically well-posed in $\mathcal{H}_k^s$ for any $s > s_k^*$ while the bilinear estimate, the key for the proof of the analytical well-posedness, fails if $s < s_k^*$.

  For fixed k, viewing the above critical index s_k^* as a function of the coefficients a_1, a_2, (c_{ij}) and (d_{ij}), its range $\mathcal{C}_k$ is called the *critical index set* for the analytical well-posedness of (1) in the space $\mathcal{H}_k^s$. By invoking some classical results of Diophantine approximation in number theory, we identify that

$$\mathcal{C}_1 = \{-\frac{1}{2}, \infty\} \cup \{\alpha : \frac{1}{2} \leq \alpha \leq 1\} \text{ and}$$
$$\mathcal{C}_q = \{-\frac{1}{2}, -\frac{1}{4}, \infty\} \cup \{\alpha : \frac{1}{2} \leq \alpha \leq 1\} \text{ for } q = 2,3,4.$$

This is in sharp contrast to the $\mathbb{R}$ case, where the *critical index set* $\mathcal{C}_0$ for the analytical well-posedness of (1) in the space $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ consists of exactly four numbers: $\mathcal{C}_0 = \{-\frac{13}{12}, -\frac{3}{4}, 0, \frac{3}{4}\}$.

- 16h00-17h00: Christophe Bourel (LMPA, ULCO)

  Title: Pollutant transport in shallow aquifers.

  Abstract. In this work, we are interested in the modeling the water flow in shallow aquifers. In particular, we are concerned with the contamination of the water table by pollutants flowing from the surface (e.g., from fertilized fields) under the influence of rain. In this context, it is crucial to accurately describe the subsurface flow in the whole aquifer, i.e., in the saturated water table and in the unsaturated vadose zone. In the latter, which is rich in oxygen, the contaminants transported from the surface continue indeed to undergo chemical reactions that can change their nature. This type of subsurface flow is classically described by the 3d-Richards model, which is known to be very difficult to handle numerically, especially in the considered situation of a large geometry and over long time periods. To avoid these difficulties, we exploit the shallow geometry of the aquifer to provide numerically efficient alternatives to the 3d-Richards model. In a first part of the presentation, we show that in shallow aquifers, the flow admits a fast component characterized by 1d-vertical Richards problems and a slow component described by a reduced 2d-horizontal problem in which the hydraulic head is constant with respect to the vertical variable. Second, we couple these types of flows to derive two new models that approximate the Richards model (they admit the same dominant components). These models models allow for a numerical treatment that reduces the high computational cost associated with the Richards model. Third, we adapt the strategy to propose an associated transport model in order to describe the pollutant transport.

19h30: Dinner

Program: June 2

- 09h30-10h30: Jie Shen (Purdue University)

  Title: The SAV approach for dissipative and conservative nonlinear systems.

  Abstract. I will present some recent advances on using the scalar auxiliary variable (SAV) approach to develop highly efficient and accurate structure preserving schemes for a large class of dissipative and conservative nonlinear systems. These schemes can preserve energy dissipation/conservation as well as other global constraints, only require solving decoupled linear equations with constant coefficients at each time step, and can achieve higher-order accuracy. In particular, I will discuss applications of the SAV approach to incompressible Navier-Stokes equations.

- 11h00-12h00: Paul Vigneaux (LAMFA, UPJV)

  Title: Shallow Water models for viscoplastic flows.

  Abstract. We present some developments on the Saint Venant equations from the Navier-Stokes-Bingham model. Specific finite volume schemes are designed to deal with the yield stress of the constitutive law. Comparisons with dam-break physical experiments will also be discussed.

12h30-14h00: Lunch

- 14h00-15h00: Ahmed Ratnani (Université Mohamed VI Polytechnique, Ben Guerir)

  Title: TBA.