Abstract

Complexities of hypercubic billiard words

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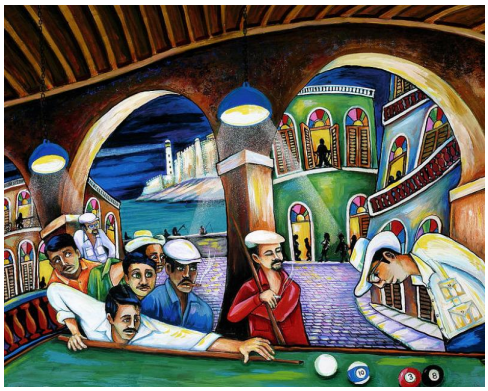
Based on a joint work with Léo Vivion.

Sturmian words (1940) form a class of infinite words over the binary alphabet which sheds light on the remarkable interactions between combinatorics, dynamical systems and number theory. These interactions are reflected in the various ways to define them. For instance, Sturmian words are equivalently

- words with *factorial complexity* $n+1$, i.e., admitting exactly $n+1$ factors of length n for all n (a *factor* of w of length n is a subword of w written with n consecutive letters);
- binary aperiodic words with *imbalance* equal to 1: all factors of a given length contain, up to a difference of one, the same numbers of 1s (and thus as well, the same numbers of 2s);
- the symbolic trajectories of a ball in a square billiard, launched with a momentum with rationally independent entries.

They give birth to several generalizations on the $d-$letter alphabet for $d \geq 3$, depending on the considered definition (e.g. : Arnoux-Rauzy or episturmian words, words associated with other $d-$dimensional continued fraction algorithms, polygonal or cubic billiard words, etc.) A large program, initiated by Rauzy in the 80s, is to determine which properties are still equivalent in higher dimension, and which are not.

In this talk, I will focus on two combinatorial quantities, the factorial and abelian complexities, of words generated by a billiard in a cube of dimension d.



Pepitos billiards, Arturo Cisneros, 2011.